

# Smile!

## ASSESSMENT #HSMA18

### Benchmark Grades

9–12

### Summary

Students use math skills to investigate quadratic relationships.

### Keywords

Algebra

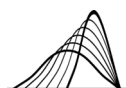
Applying mathematics to the real world

Curve of best fit

Data

Graphs

Quadratic equations



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## ASSESSMENT #HSMA18

### Information for the Teacher

#### Task Description

##### Task 1:

Students collect a set of quadratic data by graphing the shape of their dental arch (teeth).

##### Task 2:

Students collect data from five other people and present the data on separate graphs.

##### Task 3:

Students look for an average dental arch by finding the curve of best fit for the data. They also devise and implement a method for identifying an upper and lower bound for the data.

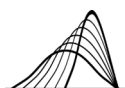
##### Task 4:

Students devise and implement a method for finding an average length of the dental arch.

For students who finish Tasks 1 through 4 ahead of the class, there are two enrichment tasks provided at the end of the assessment.

#### Required Materials

Graph paper  
Graphing calculator

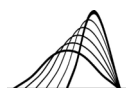


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Assessment Introduction: Here is what you will do ...

In this assessment you will be working with a parabola that you carry around with you at all times—your teeth! Your teeth create what is called the dental arch or the dental arcade. Lots of people are interested in the dental arch besides dentists and orthodontists, such as the people who produce teeth guards for athletes.

Your job in this assessment is to express the average shape and length of the dental arch. You will collect and analyze data about people's teeth.



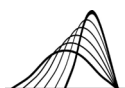
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## Task 1: Collect a Set of Data

Your job is to graph your teeth.

- First, you must take an impression of your bite. Devise a way to do this. In a paragraph, describe your method.
- Implement your method for getting a bite impression of your teeth. Once you have your impression, draw a parabola through the middle of the bite impression.
- To graph your impression, you need to have the parabola on graph paper, so trace the parabola of your bite impression onto a sheet of graph paper.
- Create a set of axes for the parabola so that the y-axis is the axis of symmetry and the vertex of the parabola lies at  $(0, 0)$ .
- Choose eight to ten points on your parabola that you can estimate the coordinates of and record the coordinates on the graph. Two of your points should be the points that represent the ends of the dental arch.

Recheck your work for errors in spelling or grammar.



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## Task 1: Scoring Guide

### 4 Exemplary

- ❑ Criteria for the “Proficient” category have been successfully completed.
- ❑ Advanced work is included. For example, the student finds the curve of best fit for the bite impression or presents research about the dental arcade. Other examples include:

### 3 Proficient

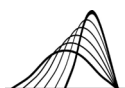
- ❑ The procedure for taking a bite impression is clearly described and is in a correct sequence.
- ❑ The parabola of the bite impression is neatly drawn on graph paper.
- ❑ The axes are well-chosen and clearly identified.
- ❑ Eight to ten points on the parabola are identified and their coordinates are estimated.
- ❑ Spelling and grammatical errors do not significantly affect the communication of ideas.

### 2 Progressing

- ❑ Four of the criteria for a “Proficient” score are met.
- ❑ More work is required.

### 1 Not meeting the standard(s)

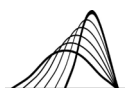
- ❑ Fewer than four of the criteria for a “Proficient” score are met.
- ❑ The task should be repeated.



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## Task 2: Collecting More Data to Search for the Average

To find an average or general shape and length of the dental arch, more data must be collected. Your job is to collect data from at least five people, completing the same procedure that you did for Task 1. For each person that you collect data from, you should have a graph of the parabola of the bite impression that has eight to ten points identified and the coordinates estimated. Orient the axes in the same way that you did for Task 1. Label each graph with the name of the person from whom you collected the data.



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## Task 2: Scoring Guide

### 4 Exemplary

- ❑ Criteria for the “Proficient” category have been successfully completed.
- ❑ Advanced work is included. For example, the student gathers additional data or suggests a process for finding the average shape and length. Other examples include:

### 3 Proficient

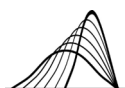
- ❑ Five graphs neatly and accurately present the data collected.
- ❑ On each graph, eight to ten points are identified and the coordinates are estimated.
- ❑ The axes are correct and identified.
- ❑ Each graph is labeled with the name of the person from whom the data was collected.

### 2 Progressing

- ❑ Three of the criteria for a “Proficient” score have been met.
- ❑ Additional work is required.

### 1 Not meeting the standard(s)

- ❑ Fewer than three of the criteria for a “Proficient” score have been met.
- ❑ The task should be repeated.



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## Task 3: Finding the Average

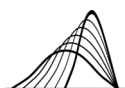
To get a clear picture of the average, all of your data must be on the same graph. Create a new graph on which you graph the eight to ten coordinates from each of the graphs you created in Tasks 1 and 2. Do not trace the parabola from the original graphs. Create the graph so that it is clear which points belong to which person.

Now you have a lot of data that you can use to find the average shape or the curve of best fit for all the data. Use a graphing calculator or computer software to calculate the equation of the curve of best fit. Record the equation on your graph and then draw the equation on the graph.

Knowing the average shape is helpful, but it is also useful to know how much variance there is from the average shape. One way to communicate the variance is to determine the parabola that could be described as the “upper bound” and a parabola that could be described as the “lower bound” of the data.

Create a procedure for identifying an upper and lower bound for your data. Write a paragraph describing your procedure. Implement your procedure and identify the equations of the upper and lower bounds and graph the equations.

Recheck your work for errors in spelling or grammar.



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## Task 3: Scoring Guide

### 4 Exemplary

- ❑ Criteria for the “Proficient” category have been successfully completed.
- ❑ Advanced work is included. For example, the student graphs the data on the computer or calculates the upper and lower bounds using more than one method. Other examples include:

### 3 Proficient

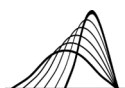
- ❑ The graph of the data is accurate and each point is clearly identified with the person from whom the data was collected.
- ❑ The equation for the curve of best fit is calculated and graphed correctly.
- ❑ The procedure for identifying the upper and lower bounds is described well and is correct.
- ❑ The procedure for identifying the upper and lower bounds is implemented.
- ❑ The equations of the upper and lower bounds are identified and graphed accurately.
- ❑ Spelling and grammatical errors do not significantly affect the communication of ideas.

### 2 Progressing

- ❑ Four or five of the criteria for a “Proficient” score have been met.
- ❑ Additional work is required.

### 1 Not meeting the standard(s)

- ❑ Fewer than four of the criteria for a “Proficient” score have been met.
- ❑ The task should be repeated.

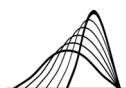


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## Task 4: Find the Average Length of the Dental Arch

To create orthodontic equipment as well as teeth guards for athletes, the average length of the dental arch is important. Devise a procedure to find the average length of the dental arch. Describe this procedure in detail. Implement your procedure to find the average length of the dental arch. Show any calculations required in your procedure.

Recheck your work for errors in spelling or grammar.



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## Task 4: Scoring Guide

### 4 Exemplary

- ❑ Criteria for the “Proficient” category have been successfully completed.
- ❑ Advanced work is included. For example, the student implements more than one procedure and compares the differences or creates a particularly well thought out and thorough procedure for finding the average length. Other examples include:

### 3 Proficient

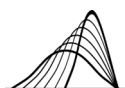
- ❑ The procedure for taking a bite impression is clearly described and is in a correct sequence.
- ❑ The described procedure is implemented with all supporting work shown.
- ❑ Spelling and grammatical errors do not significantly affect the communication of ideas.

### 2 Progressing

- ❑ Two of the criteria for a “Proficient” score have been met.
- ❑ Additional work is required.

### 1 Not meeting the standard(s)

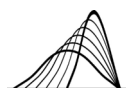
- ❑ Fewer than two of the criteria for a “Proficient” score have been met.
- ❑ The task should be repeated.



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## Enrichment Task 1: Comparing Your Findings to Others

Your study was done with a fairly small pool of data. Compare your findings with someone else who has also completed this assessment. Your job is to compare the average shape you found to someone else's. Write a report in which you describe your procedure and show the results and supporting work when you implemented the procedure.



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## Enrichment Task 1: Scoring Guide

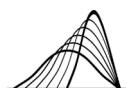
Here is your opportunity to decide what makes up a good response. Think of what your response should include to be proficient.

**4 Exemplary**

**3 Proficient**

**2 Progressing**

**1 Not meeting the standard(s)**



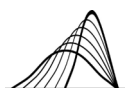
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## Enrichment Task 2: Comparing Your Findings to Published Findings

Dr. Paul Sampson did a study similar to yours with a data pool of sixty-six dental arches. His procedure was a little bit different. When he graphed each dental arch, he created the graph so that the ends of the dental arch always fell at (0, 0) and (1, 0). The average dental shape was described by the following equation:

$$0.957015(x^2 - x) + 0.007691xy + 0.254125y^2 + 0.139585y = 0$$

Your job is to find a way to compare your average shape to Dr. Sampson's. Write a report in which you describe your procedure and show the results and supporting work when you implemented the procedure.



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## Enrichment Task 2: Scoring Guide

Here is your opportunity to decide what makes up a good response. Think of what your response should include to be proficient.

**5 Exemplary**

**4 Proficient**

**3 Progressing**

**1 Not meeting the standard(s)**

